Power System Analysis Notes

GUR person to reach out to:  
Anthony Carroll, Energy Delivery Manager, [carrolla1@gru.com](mailto:carrolla1@gru.com)

Rene Zamot, Puerto Rican guy, likes sustainability, [zamothr@gru.com](mailto:zamothr@gru.com)

Cynthia Blunt-Turner, ECE department, [cturner@ece.ufl.edu](mailto:cturner@ece.ufl.edu)

Look for full bright and similar scholarships

PhD student, my old TA: Nader Aljohani, [eng89nader@ufl.edu](mailto:eng89nader@ufl.edu)

# Ch 1, State Estimation of Electric Power Systems

## Historical Context

The field of power **Security Control** or **Real-Time Operation** started in the late 1960s. It was motivated by the east coast blackout and enabled by recent advancements in technology.

The current and power follow can be derived from measurements of **complex voltage**, and complex voltage is therefore defined as the **EPS state variables.**

**EPS variables** measured at nodes around the power grid and need to be compared with each other to give information about the state of the grid. Local readings are therefore transmitted to control center where they are filtered and analyzed by computer programs (this is the **PSSE process**).

The **PSSE** process need to keep evolving to correspond with the development of the grid (more advanced measurements)

## The Process of PSSE

The PSSE process consist of 4 steps:

1. **System Topology**  
   In this step the system topology is established though establishing the correct location of the meters in the **bus-branch model.** This relays on the information of system switches and circuit breakers and physical layout of the network.
2. **Observability**  
   The available measurements are evaluated to see if they allow for the observation of the entire system. If this is not the case **pseudo measurements** can be used to restore the observability of the system.
   1. What are the drawbacks of **pseudo measurements**?
      1. Pseudo-measurements are based on data form the database of the operation centers such as load forecasting, generation forecast, historical data, etc...
         1. This means it is not based on estimates rather than current/real information
         2. Pseudo measurements will not be accurate for unforeseen events
3. **State Estimation**We mainly look at using static state estimation which allows us to disregard time and rater look at a “photograph” of the state of the system at a given time. There are multiple approaches for state estimation, but the most popular method is the Weighted Last Square (WLS) method
   1. State estimation can be done both statically and dynamically
      1. This is kind of time domain V.S. frequency domain?
4. **Gross Error Analysis/Processing**In this step **gross errors** (GEs) are identified and eliminated before moving back to step 3 and performing the state estimation without the effect of the GEs. This process is continued until no more GEs are observed.

**What are residuals?**

* A residual is the result of subtracting the real measurements (or true values, not measured?) form the estimated measurements. They are indirectly the result of the state estimation.

# Ch 2, Real Time Operation of Power Systems

This chapters gives an overview of the 4 steps of PSSE with a focus on system topologies (step 1).

## Operating States of an EPS

The operation of an EPS must happen within a set of 3 constraints

* **Load Constraint**The power/current input must be equal to the power/current output (Kirchhoff's 1st law)
* **Operating Constraints**  
  The operation limitations of the system equipment must not be exceeded.
  + E.g. maximum current, voltage and power ratings for a transmission line or transformer
* **Security Constraints**The system’s ability to tolerate isolated failures without resulting in the entire network failing. This constraint contains a list of contingencies that the system should be able to handle. However, this list may not include some contingencies which would then not be protected.

These operating constraints are mathematically represented by the 3 equations seen below:

|  |  |
| --- | --- |
|  |  |
| Load Constraint |  |
| Operating Constraint |  |
| Security Constraints |  |

The variable “x” is the vector describing the complex bus voltage of the system.

There can be different constraints at specific lines.

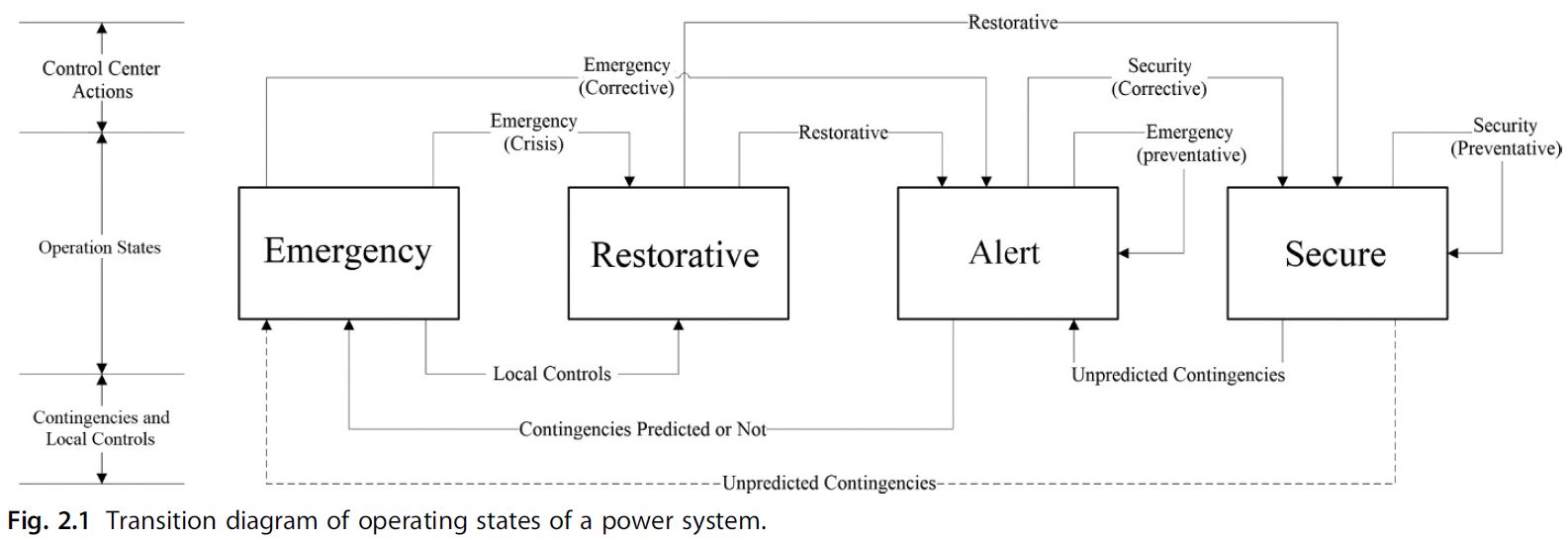
### Operational States

These constraints give 4 possible operation states of the system

1. **Normal Secure State**  
   There are no constraint violations, and the system can handle the occurrence of any listed security contingencies without falling into a state of emergency.
2. **Normal Alert State**  
   One of the listed security contingencies has occurred, but there is no violations of the Load or Operation constraints. In this state, the occurrence of an additional listed security contingency could cause a state of emergency
3. **State of Emergency  
   At least one** operation constraint is beeing violated. This state is caused by a combination of listed contingencies, or a severe unforeseen contingency. **Emergency eliminations** can be done.
4. **Restorative State**Operation constraints are satisfied, but the system is not intact (one or more sections of the system has been disconnected and is not operational). This occurs when an integrity of the system must be sacrificed to resolve an emergency.

The goal is to keep a system in a normal state and avoid the emergency and restorative state as much as possible. Transitions between the states are categorized as **voluntary transitions** (control actions) and **involuntary transitions** (system disturbances)

|  |  |  |
| --- | --- | --- |
| Voluntary | Secure -> Secure | The system is modified so that it can better withstand a future load change that has been predicted by a short-term load forecasting program (could forecast ~1 hour) |
| Secure -> Alert | The control center acts in a corrective mode to optimize the system operation.  A security constraint is intentionally violated to optimize the system operation |
| Alert -> Secure | Control measures are adopted to return the system to the secure state. |
| Emergency -> Alert | Emergency controls work in a corrective mode through, for example, re-dispatching generators, varying control voltages or transformer taps, switching capacitor banks, etc. **This does not sacrifice the system integrity**. |
| Emergency -> Restorative | This occurs when the control center cuts of a section of the grid and is done when it is not possible to resolve an emergency through other means. **This sacrifices the system integrity.** |
| Alert -> Alert | The emergency control function is used in a **preventive mode** to change to operative point of the system to avoid an anticipated emergency. |
| Restorative -> Alert  or  Restorative -> Secure | The restorative control is used to reconnect previously disconnected sections of the system in a way that will bring the system back to an alert of secure state |
| Involuntary | Secure -> Alert  or  Secure -> Emergency | Unforeseen violation of a security contingency or a regular load increase causing the violation of a security constraint |
| Alert -> Emergency | An additional listed or unforeseen contingency occurs when the system is already in the alert state |



## Network Analysis System

According to Dy-Liacco, 1974, security strategies are divided into **Security monitoring**, **security analysis**, and **preventive control**:

* **Security Monitoring**  
  The system topology and current operation condition is monitored and evaluated for possible violations of operation constrictions
* **Security Analysis**The real time operation state of the system is analyzed (statically or dynamically) and compared to predetermined contingencies and possible further operation trends
* **Preventive Control**If the security analysis indicates that a contingency my occur preventive actions will be taken to improve the security of the system

These are the main function of the network analysis program:

* **Pre-filtration Program**

A **compatibility test** is performed on the system measurements and used to determine obvious errors (not GEs). This is a relatively trivial test which looks for obvious errors such as negative voltage magnitudes, deviation of multiple magnitudes form expected values, or large differences between the input and output current of substation nodes.

* **The PSSE Process**

This process is described in chapter 1 and is, in short, used to eliminate GEs and obtain the current operation state of the EPS

* **Load Forecasting Program**

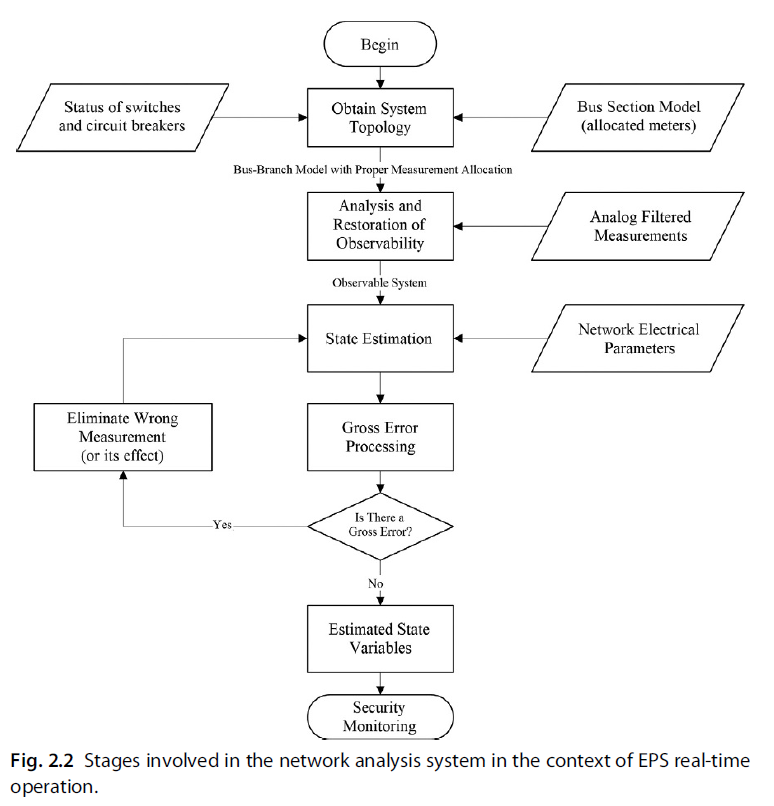
This program uses demand forecasting to predict the future changes of the EPS and adjust power generation and the system structure accordingly

* **Power Flow**

The internal system topology and the load forecast of the external bus is used to determine the complex voltage of all the busses of the internal network and the external network connections. This is done through the PSSE process and can also predict how the system will respond to future changes.

* **Security Analysis Program**  
  Subprograms are used to evaluate possible contingencies based on the current operation state of the EPS. This is generally done statically because dynamic evaluation is too computationally heavy.
* **Optimum Power Flow**

This the determination of how the system can best be controlled to optimize load, security, the operation point, and operation costs.



## State Estimation in Electric Power Systems

This is a more extensive description on the PSSE steps

### Step 1, Topology, and meter configurations

This step consists of consulting a database that contains the static configurations of the EPS and logic measurements which gives the status of switches and circuit breakers (switching devices).

* Busses that have the same voltage level can be interconnected by switches and circuit breakers and form a single bus.
  + Is a bus diagram a simplified diagram of the power system as seen in figure 2.4
* Different switching device configurations will give different network topologies.
* The system must process all circuit components to obtain the network configuration, and this will have to be done again if the switching device configurations change.

Diagram, schematic

Description automatically generated Diagram, schematic

Description automatically generated

This is the general network configuration prcess for state estimationns:

1. **Substation Configuration**

Bus sections are determined by identifying which sections are connected with switches.

1. **Network Configuration**

The outward connections of the previously defined bus sections are traced and confirmed to connect, in series, to transmission lines and transformers

1. **Association of metering devices**

Decides which meters are connected to which bus sections and which meters should be used to obtain readings.

1. **Result Tabulation**

This is the construction of the bus branch model from the information found in steps 1-3

Initially, all these steps were performed sequentially for every change that occurred in the network. However, this unnecessarily wastes computational power, and the method has therefore been improved. A recent network configuration technique uses **Node depth representation** and has the following advantages

* Simultaneously performs substation and network configuration
* Direct association of network components and meters
* Does not need to redo the entire network configuration when a single change occurs
* Indicates the necessary changes in the **Jacobian matrix** of the weighted least square estimation after status changes of switching devices which allows **partial matrix factorization methods** to perform the matrix state estimation faster.

### Step 2, Observability analysis and restoration

This step uses the topological information from step 1 to determine which sections of the electrical system is directly observable. If the entire network is not observable, we end up with **observable islands**. There are various techniques for extrapolating to the non-observable sections of the network and these will be covered in **chapter 5.**

### Step 3, State estimation

This step aims to determine the state variables of the EPS through the topology and measurement information from step 1 and the pseudo-measurements from step 2.

There are multiple state estimation techniques, and the most widely used one, weighted last squares (WLS), will be covered in **chapter 4**.

### Step 4, Gross error elimination

This step aims to eliminate measurement errors stemming from:

* Analog to digital conversion errors
* Telecommunication channel errors
* Lack of measurement synchronizations
* Saturation of measurement transformers

Most GE elimination techniques are based on WLS state estimation and will be covered further in **chapter 6**.

## Influence of topological and parameter errors in the PSSE

Topology errors (TE) form step 1 and parameter errors (PE) from step 2 results in incorrect assumption for the following steps. This cause increases in the **residuals** of analog measurements and results in measurements beeing wrongly identified as GEs and being eliminated in step 4 and reducing the system redundancy to 0.

## Synchronized phasor measurements in the PSSE process

SPM aims to improve the reliability of the PSSE process.

While SCADA measurements are taken every few seconds the SPM relies on PMU measurements of both voltage and current which are taken between 20 and 60 time every second and synchronized through the GPS clock signal.

However, it is expensive to implement PUM units and they will likely not replace SCADA measurements soon.

There is work beeing done in creating hybrid systems that can combine SCADA and PMU measurements, and these systems give good results under steady state conditions. However, the different time scale of SCADA and PMU results in low accuracy when changes occur in the network. Forecasting-aided state estimation (FASE) is now attempting to address this issue.

|  |  |  |
| --- | --- | --- |
|  | Non-SPM | SPM |
| Type of measurement | SCADA | PMU |
| Sampling rate | Seconds | Milliseconds (20, 30, or 60 per second) |
| Analysis | Nonlinear analysis | Linear analysis (A(x)=B, would be faster to solve) |
| Measurement method | Only Static measurements | It technically enables dynamic measurements, but this is not the case. |
| Current state | Cheap and widespread in most grids | Expensive telecommunication requirements |
| Comparison table | | |

# Ch 3, Power Flow in Electrical Systems

The calculation of power flow generally addresses two scenarios:

* The connection between a bus and the ground
  + E.g.: generators, loads, capacitors, reactors
* Connections between two bus networks
  + E.g.: transmission lines and transformers

## 3.2 Basic Problem Formulation

Table

Description automatically generatedFor power flow problems we work with the variable seen below, and depending on the type of bus, two variables are measured, and the remaining two variables are calculated

* : Voltage magnitude at bus k
* : Voltage phase angle at bus k
* : Net active power (generation minus load) at bus k
* : Net reactive power at bus k

The Vθ bus (or reference, slack, or swing bus) is generally assigned to a large generator and provides the angular reference for the entire power system (for the classical centralized solution there is **only one** reference bus). This bus is also used to balance the power demand and supply of the system and its active power is not initially justified/identified.

* The bus has

|  |  |  |  |
| --- | --- | --- | --- |
| Type | Given | Unknown | Description |
|  |  |  | * Also called generator, reference, slack, or swing bus * For classical analysis there is only bus one per system. This bus is placed on the **main generator**. * Used to balance the system |
|  |  |  | * Also called regulated busses * These are generally also **generator busses** * Can also have a **Sync. Condenser** instead of a generator |
|  |  |  | * Also called **load busses** * These are placed on busses that directly feed loads |

Diagram

Description automatically generatedAll power flow equations essentially shows that the active and reactive power that enters a bus, k, (Pk, and , respectively) is the same as the active and reactive power that leaves it ( and , respectively). As seen below, and are functions of the voltage magnitude and angles of the k bus ( and ) and each of the network busses, l ( and ).

Assumptions:

* **Net power injections** are positive when they enter the bus (generation) and negative when they leave it (load).
  + This is used for the generators that are connected to sub stations.
  + This is also used to loads
  + E.g.: connection between a bus and the ground
* **Power flows** are positive when they exit a bus and negative when they enter it.
  + This is between two sub stations through equipment along the way.
  + E.g.: transmission lines and transformers
* For the shunt elements of the buses use the **injections convention**.

There are also some constraints of the nodal voltages for PQ busses and the reactive power injection for PV busses which are a part of solving the power flow problems.

Example:  
If you have a PV bus with a known power injection and voltage level. The constraints are the physical limits of the generator. If a solution converge tovalues ouside of the sonctraint, this solution is not valid.

## 3.3 Modeling and Equations for Calculations of Power Flow

### 3.3.1 Equations of Power Flow between Busses

#### Chart, diagram, schematic Description automatically generatedTransmission lines

The model of a transmission line defines a TL by three characteristics:

* : the series resistance
* : the series reactance
* : the shunt susceptance

Because nodal circuit analysis is used, we need to transform the series impedance, , to an equivalent admittance,:

Then, the current leaving each bus can be found:

**Note:** E is used to describe vectors while V is used to describe scalar values of the voltage, so:

Now we know both the current and voltage value at each of the terminals, k and l, and the flow of real and reactive power can be calculated through some heroic algebra (or just using a calculator)

Text, letter

Description automatically generated

Seperating the real and imaginarry terms this then gives us the real and imaginary power at kl:

Text

Description automatically generated

And knowing that and , we find the values at lk:  
Text

Description automatically generated with medium confidence

**Note:** and

#### Transformers

Transformers are generally modeled using an ideal transformer with a turn’s ratio of (which can be a complex number) and a series impedance that represents the non-idealities of the transformer. A reference point, , is also located between the ideal transformer and the series impedance. This reference point does not really exist, but it is useful for completing a step of the calculations without considering the series impedance. Diagram, schematic

Description automatically generated

* : the (complex) voltage magnitude at point ,
* : the series impedance of the transformer
* : the turn’s ratio of the transformer (can be a complex number!! )
* : the inverse of the turn’s ratio of the transformer
* : absolute value of the turn’s ratio (never a complex number)
* the value of the lag/phase shift caused by the transformer (in rad)

There are multiple types of transformers that are defined based on their tuns ratios and whether they will phase shift the power that goes through them (if is complex or not).

* Transmission line
  + There is a shunt capacitance, but no voltage/current transformation and no phase shift
* In-phase transformer
  + There is a voltage/current transformation but to shunt capacitance or phase shift
* Phase shifting transformer
  + There is a voltage/current transformation and a phase shift, but no shunt capacitance
* Purely phase shifting transformer
  + There is a phase shift, but no voltage/current transformation or shunt capacitance

|  |  |  |  |
| --- | --- | --- | --- |
| **Device** |  |  |  |
| Transmission Line | 1 | 0 | Any value |
| In-Phase Transformer | Any value | 0 | 0 |
| Phase Shifting Transformer | Any value | Any value | 0 |
| Purely Phase Shifting Transformer | 1 | Any value | 0 |

##### Transformer taps

Transformer taps are used to slightly vary the turns ratio of the transformer (usually 5% up and down in increments of 2.5%). This is done to compensate for the fact that the transformers input terminal voltage is often not the exactly the nominal voltage.

**Notes:**

* Taps can only be changed when the transformer is de-energized
  + Tap configuration is set to account for the average system voltages, not minor fluctuations
* There are taps on the both the high and low-voltage side
* Tap changing under load (TCUL) transformers, also called voltage regulators, can change their taps while under loads

#### Generalized π model

The generalized model gives a set of equations that can be used to determine the power flow between a k and l terminal that are separated by either a transmission line or any of the transformer types. If the relevant constant values given and seen in the table above are used any irrelevant terms will cancel out.

For k to l:

For l to k:

Chart, diagram, schematic

Description automatically generated

### Diagram Description automatically generated3.3.2 Bus Power Injection Equations

|  |  |
| --- | --- |
|  | Reactive power injected to the k bus from the shunt susceptance |
|  | Current injected to the k bus through the shunt |
|  | Active power traveling along the k-l branch |
|  | Shunt susceptance is **positive** (capacitive) |
|  | TL susceptance is **negative** (inductive) |
|  |  |
|  |  |

The net amount of current that goes into a generic bus is the sum of the input current, , and the shunt current,

This input is then set equal to the sum of the current going to all the bus outputs:

: the **node(s) adjacent** to , : the set of busses adjacent to k : # of busses in the system

**Note:** is defined as going from ground to bus k and is therefore given by

The generalized model gives this expression for the current flow from bus k to bus l:

By summing up the for each network element, , and including the k shunt () we then get another equation for :

or

For large computations we will arrange this equation into a matrix form:  
I: **vector** of current injection, Y: admittance **matrix** for the system, E: **vector** of bus voltages

#### The Admittance Matrix

The admittance matrix is a square matrix with dimensions equal to the number of busses and is symmetrical along the diagonal.

The off-diagonal elements are equal to the negative admittance between and :

The main diagonal elements are equal to the sum of all the admittances connected to the bus . This is called the self-admittance:

* For large systems the admittance matrix is a sparse matrix.

Dense matrix: a matrix where most of the elements are non-zero

Sparse matrix: a matrix where most of the elements are zero

The complete matrix form of the current injection into a generic bus k (where K: is the number of busses adjacent to bus k, including itself) is:

The matrix is also generally divided into a real and imaginary matric, resulting in:

Again, a heroic amount of mind-numbing algebra and matrix math is needed to arrive to the matrix equations for real and reactive power:  
Text, letter

Description automatically generated

Isolating the real and imaginary parts:  
Text, letter

Description automatically generated  
Finally, in terms of the admittance matrix, Y, the above equations become:  
Text

Description automatically generated with medium confidence

## 3.4 Nonlinear Power Flow

The two power flow equations are seen below, and power flow problems are usually solved in two steps. These equations are nonlinear because of the cos and sin terms

1. The 1st sub system is composed of set of equations with the dimensions 2NPQ+PV of **non-linear algebraic equations** with the same number of unknowns.
   1. PQ unknowns: and
   2. PV unknowns:
   3. The unknowns of this system are given by
   4. The superscript “sp” indicates that something is a specified value
   5. For this approach the unknowns, Vk­ and are **implicit**, and requires an **iterative solution process**
2. The 2nd sub-system is solved after the 1st and its dimensions are NPV+2 of **non-linear algebraic equations** with the same number of unknowns.
   1. and are known for all busses
   2. Calculations on the reference bus (Vθ): and
   3. Calculations on PV busses:
   4. The unknowns can be **explicitly** calculated so there is no need for an iterative solution

### 3.4.1 Newton Method

1. You have a non-linear equation, , with a variable, , and you want to find the value that will result in
2. You use an initial value of as the starting point, , where the superscript denotes the iteration number and **not the power of .**
3. Linearize the function through performing a Taylor series expansion around the starting point, , and neglect the higher order terms. This results in the approximation:
4. Solving the approximation for then gives an improved guess, , for which x value will result in .
5. Step 3 is then repeated until the correction value, , is within an acceptable amount.

The Newton method can also be applied when both and are vectors. This is done through using a **Jacobina matrix**, J, to take derivatives, and **a correction vector,** , to determine how close the guessed vector, , is to the true answer for :

The correction vector is calculated through solving the equation:

And the next x-vector is then achieved through

### 3.4.2 Power flow solutions by Newton’s Method

When solving subsystem 1 through Newton’s Method the primary task is to determine the correction vector and we do this through solving:

Where:

Since and are given values, their derivative is always zero, and the Jacobian matrixes are therefore relatively easy to define:

This applies for all the terms in the Jacobian, and we thus get:

Then, rewriting in a vector format we get:

Text, letter

Description automatically generatedThe Jacobian submatrices have the variables of and and are known and seen in equation 3.64 to 3.67. Further, the diagonal elements can also be written as seen in 3.68 to 3.71:  
Text, table

Description automatically generated

#### Newton’s Method Procedure

1. We start by setting the iteration number to 0 , choosing the initial angle of the PQ and PV buses , and choosing the magnitude of the PQ bus voltages
2. Then, the real power of the PQ and PV buses and the reactive power of the PQ buses is calculated.
   1. These answers are then used to determine the correction vectors, and
   2. These are the formulas for the correction vectors:
3. Test if the and are close enough to satisfy the convergence criterion by evaluating the respective correction vectors: and .  
   If the convergence criterion are not satisfied, continue to step 4
4. Calculate the Jacobian matrix:
5. Determine the next and values:  
   1. and are calculated from this linear system:
6. Increase the number of iterations and return to step 2.

##### Questions

* Definition of the nodal admittance matrix:  
  + defines all the non-diagonal cells of the matrix
  + defines all the primary diagonal cells of the matrix
* What is the formula for the correction value?  
  + The correction vector of the specified quantity is different than the correction vector of the variable!
    - The specified value is given and will never change, but the calculation of the correction vector for the variable will change every time.

#### Newton-Raphson technique

This is the technique outlined in the book, and the detailed procedure is as follows:

1. Assemble the admittance matrix:

If using the per unit numbers, the turns ratios between different voltage levels will be 1:1. However, you still have to incorporate in the transformer tap values in the turn’s ratio matrix, .

1. Arrange specified values and variables.

|  |  |
| --- | --- |
| PQ-bus | Specified values: and  Variables: and |
| PV-bus | Specified values:  Variables: |

Variable matrix:

Specified value/constants matrix:

In MATLAB:

* + Declare variables as symbols
  + Declare constants as decimals

1. Establish the power equations

There will be 2\*NPQ+NPV equations

: Admittance angel : Voltage angle at bus k : Voltage angle at bus l

1. Compare the specified powers and the powers for the power equations.

If and satisfies the convergence criterion, we are done, if not continue

1. Calculate the next iteration of the state variables and
2. Calculate the Jacobian of the power function vectr

The main diagonal elements are equal to the sum of all the admittances connected to the bus . This is called the self-admittance:

## 3.5 Power Flow in **Distribution Systems**

Newton’s method is generally **not suitable** for calculating power flow in a distributed system. This is because it relies on **matrix factorization** that is not compatible with distributed systems. Thus, we have to us a different approaches for solving distribution systems

The **forward/backward sweep** method is most used to work with distributed systems. This method consists of two steps:

1. Start with a backwards sweep starting from the end of a feeder to the substation where the **current or power flow** of all segments are calculated. For the 1st backward sweep all node voltages are assumed to have their nominal value.
2. A forward sweep is done, and all the nodal voltages are calculated using the **currents or powers** from the previous step, the substation voltage, and the line impedance.
   1. If there is not convergence between the voltages of step 1 and 2, step the process is repeated with the new voltage values calculated in step 2

### Gauss-Seidel power flow solution

#### PQ-buses

For load busses we know both the real and reactive power and can therefore directly calculate the next iteration of the voltage at a given bus,, by:

This will give a complex voltage and be our next guess for

#### PV-buses

For regulated buses the reactive power is not specified, so we first have to calculate this:

With this, we can now calculate a temporary voltage:

However, because we already know the magnitude of the voltage at this bus, we will only hold onto the imaginary part of the temporary voltage (because the imaginary part is usually the smallest).

Then use the imaginary and magnitude of the voltage to find the real value:

The new voltage at is then given by:

### 3.5.1 Power Summation Method

Diagram, schematic

Description automatically generated

#### Procedure

1. The substation or nominal voltage is assigned to all the buses in the system
2. **Backwards sweep**. Starting at the end bus and ending at the sub station bus, the power flow of all branches is calculated using Eqs. 3.76 and 3.80  
   : apparent power entering bus , : apparent power going from bus to a load  
   : apparent power leaving bus through a downstream transmission line  
    and : real and reactive power leaving bus towards before going through   
    and : resistance and reactance of : voltage magnitude at
   1. The current running into is given by
   2. , the power being sent from k to l is given by
   3. The magnitude of is given by
3. A **forward sweep** is done and Eqs. 3.82 and 3.83 are used to update all bus voltages  
   Text, letter

   Description automatically generated
4. The voltage magnitudes of the current and the previous iteration are compared and if any of the buses have a difference greater than the tolerance the process is repeated starting from step 2. If not, the process is done, and all node voltages and power flows are determined
   1. We then use the values found in step 3, correct?

### 3.5.2 Current Summation Method

Current summation is better suited for radial systems but can also be used for weakly meshed systems.

A picture containing text, clock

Description automatically generated

#### Procedure

Note: this procedure does not consider more than one power source.

The nodal voltages are initially assumed to have the same magnitude and phase.

1. The load current of each bus is calculated by:

: the complex current demanded by bus in iteration   
: the complex voltage of in the previous iteration

: the sum of all shunt elements connected to bus

: the complex power of bus

1. **Backwards Sweep**: The current of each branch, , that connects bus with the previous bus is calculated. Here stands for current and not Jacobian:

: the set of all lines connected to bus

1. **Forward Sweep**: Starting from the substation, the voltage of each bus is calculated with the equation where the bus is one unit deeper than the bus.

: the impedance of the line between the and bus

1. It is evaluated if the difference between the and the values are sufficiently small to satisfy the convergence criteria. If it is, the process is done, if not, we return to step 2.

## 3.6 Exercise!!!

Load flow implementation (Newton Raphson)

* Do exercise 3.6
* Matlab

# Ch 4, Classical static state estimation in electric power systems

Pg. 71-108

## Relevant abbreviations

WLS: Weighted Least Square

EPS: Electrical Power Source/System

PSSE: Power System State Estimation

PMU: Phasor Measurement Units

|  |  |  |  |
| --- | --- | --- | --- |
| Equation symbols and their significance | | | |
| Symbol | Descriptions | Dimensions |
|  | Measurement vector, contains all the measurements of the system |  |
|  | Error vector, contains the difference between a measured value and the true value |  |
|  | Equation vector giving an approximation of the measured values from the state variables , |  |
|  | The covariance matrix of the measured values, the variance of each measurement is also found along its diagonal |  |
|  | Variance of a measurement, variance is the square of the standard deviation |  |
|  |  |  |

## 4.1 Introduction

Simplicity of formulation and ease of implementation in computers has made the **weighted least square** (WLS) estimation method has med the WLS the most common state estimation procedure.

This book focuses on **ultra-high voltage** and **extra-high voltage transmission** systems where the system can be assumed to be balanced, and we can therefore use the **per-phase model** (also called single phase of positive sequence model). However, this is not applicable for distributed systems which requires the **three-phase model**.

The static state estimation of EPS consists of obtaining the real time state variables of the EPS through using: the **network parameters**, **network topology**, **metering configurations**, and a snapshot of the **real time analog measurements**.

## 4.2 The measurement model

Any measurements taken in an EPS are subject to errors (inaccurate meters, communication channel errors, analog to digital conversions, etc.). A measurement is therefore always a sum of its true value, , and the corresponding error, :

, , and , can all be vectors of dimensions , where is the number of measurements.

For conventional PSSE it is assumed that error vectors have a **Gaussian distribution** with a mean of 0 and a covariance matrix . The errors are assumed to be **uncorrelated**, and R is thus a diagonal matrix with the variance of each measurement error along its diagonal (this is usually a function of the accuracy and scale of the meter).

Based on these assumptions and updated measurement function can be given by:

state variable vector (the phase angle and magnitude of bus voltages) : error vector  
: vector with equations relating the state variables and the measurements

All these vectors have dimensions of , where , and n is the number of buses in the system.

Most measurements are still taken with SCADA, but research is beeing done with moving to PMS and synchronized phasor measurements (SPM)

## 4.3 WLS state estimator

What is the WLS method   
<https://online.stat.psu.edu/stat501/lesson/13/13.1>

**This is a state estimation method that iteratively finds the state variable values and that uses the variance of each measurement to determine how much this measurement should contribute to each iteration/correction.**

### 4.3.1 Coupled state estimator

To obtain the best estimation of the state variables, , we will try to minimize the index given by:

This is achieved when , and can be expressed as , where is the Jacobian of

Gain matrix:

#### Diagram Description automatically generatedProcedure steps

1. Set the iteration index to 0 , and chose the initial x values (typically a flat start)
2. Establish and calculate , , and at
3. Get the correction vector, , of the state variables, , through the normal equation:
4. Test for convergence by . If true, stop. If false, update the state variables and iterate and return to step 2:

Note: for this state estimation we can select the bus with reference angle . This is done by:

* Introducing a pseudo measurement of at this bus (with a low -value)
* Removing the column of the Jacobian matrix that correspond to this angle. Thus, the reference angle, , will not changed from its initial flat start condition of

#### Example

Page 82

: phase angle introduced by transformers.

: phase angle of voltage at a bus or the difference between two busses

### 4.3.3 Fast decoupled state estimator

The coupled state estimation is relatively computationally heavy because it has to repeat the calculation of the gain matrix (G) for each iteration. Fast decoupled state estimation gets around this by not calculating a new Jacobian, H, for each iteration.

The process is further simplified by only considering the most relevant subsets of the Jacobian matrix.

and have the least significant impact and are therefore neglected, while and are calculated and kept constant during the iterative process.

### 4.3.4 Linear WLS state estimator (DC estimation)

This method is rarely used but provides a simplified and useful insight in power flow estimations.

For these calculations TL losses are neglected and the voltage thus remains 1 pu throughout the system. This gives rise to the following power equations:

## 4.4 Alternative WLS estimator formulations

Because the WLS estimator based on the normal equation (Eq. 4.15) is prone to poor numerical conditioning.

There are a lot of metods aimed at improving the numerical robustness of the WLS method. A lot of these are based around evaluating the **condition number of the matrix**, , through various norm calculations:

The condition number indicates the numerical robustness of a matrix

Conditions when the conditioning of the gain matrix, G, can be effected:  
Text

Description automatically generated

### 4.4.1 WLS estimator with equality constraints

?? what is the pivot??

Busses with zero net power injection (transfer buses) are defined as virtual measurements. Because a measurement of 0 power does not have any error (variance), these measurements need special treatment during analysis (any measurement with 0 variance would make the matrix go to infinity).

There are 2 typical solutions to this:

1. Assign the virtual measurements a high weight in . However, this can cause issues with
2. Represent the virtual measurements as equality constraints. This will help reduce numerical problems since we do not have to deal with the issue of variance.

With the 2nd approach we need to minimize , given the constraints

: the vector of nonlinear equation of zero power injection (virtual measurements)

The Lagrange formula is the used to combine the and into one formula which is easier to solve for computationally:

: a vector of Lagrangian multipliers.

More details in chapter

### 4.4.2 Hachtel augmented matrix method

This method considers the residuals, as unknowns and its equations is treated as a constraint:

: residual vector : weighted matrix of **remaining** measurements

This gives the Lagrangian:

: Lagrangian multiplier of the virtual measurements  
: Lagrangian multiplier of the residual constraints

The set of the 1st partial derivatives of gives:  
Text, letter

Description automatically generated

Which shows that and can thus be removed form the solution of the iterative equation:  
Text

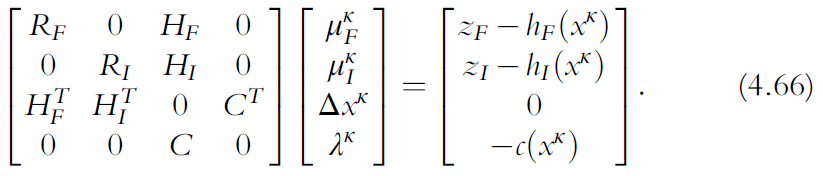
Description automatically generated with low confidence

There is something about a 2X2 pivot

### 4.4.3 Sparse Tableau method

The Sparse Tableau method is effectively a special case of the Hachtel method where the measurements are divided into two sets:

* Power flow and voltage magnitudes, set F
* Power injection measurements, set

This gives the system of equations:  


“This ordering reduces the occurrence of so-called fill-ins, that is, of the nonzero elements that arise during the factorization of the coefficient matrix.This directly influences the accuracy of the solution of the system of equations, since it brings as a benefit the reduction of possible numerical errors from the factorization.”

### 4.4.4 Orthogonal transformation method

Pg. 96, review if relevant

This method uses different weighted Jacobian and vectors and some slightly different matrix math.

This gives the normal equation:

…

This method is attractive for **practical** and **real-time applications**.

## 4.5 WLS estimator considering SCADA measurements and SPMs

Hybrid state estimator proposed by Korres and Manousakis (2011):

1. Does not require many changes of the conventional formulation of PSSE process by WLS
2. Does not require a bus to be chosen as an angular reference
3. Enables prosessing of **gross errors** in SCADA and SPM if the redundancy is adequate

The phase angle of the voltage is known for all SPM measurements. Therefore, we can no longer arbitrarily assign a zero-angel bus. Instead, all angles that must be calculated (those at SCADA buses) will be calculated from the angles measured at the SPM buses.

Differences between regular WLS and hybrid:

* Adds a known voltage angle for all SPM measurements. This will add another column to the Jacobian matrix for each new voltage angle
* Complex numbers are expressed rectangularly rather than polarly

## 4.6 Statistically robust state estimators

WLS state estimation works well when there are no gross errors. However, it quickly fails if gross errors are not identified and eliminated.

### 4.6.1 WLMS stat estimator

What is the difference between and ?

WLMS: Weighted Least Median of Squares

: vector of state variables

: median position of weighted squared residuals

: weighted residual, generated form :# of available measurements

: ith diagonal element of the covariance matrix

:#number of sample sets :# of measurements of each sample set and # state variables to be estimated

**Median**: the middle number when a set is arranged in increasing order (the average of the two middle numbers if the set has an odd number of elements)

**Mean**: the average of a set of numbers

Procedure:

1. sets of measurements are taken of a system. Each set contains measurements, which assures that the entire system is observable.
2. The state variable vector is calculated for **one** of the sets by using the **load flow method**
3. The weighted residuals,, of the measurements in the system are calculated based on the state variable vector, , form step 2.
   1. The square of each weighted residual is also calculated, and the residuals of the set are ordered incrementally
4. The position of the median of the residual, , is calculated through
5. The WLMS estimate is the calculated from the set of measurements that has the smallest squared weighted residual.

## 4.7 State estimators for distribution systems

Distribution systems are the lower voltage section of the power grid that delivers power to the consumers. Distribution systems have traditionally been radial systems and many feeders are unidirectional. The operation state of these feedres generally based on load profiles created form a composite of the different consumer load profiles and is therefore only a rough estimation.

However, with the emergence of smaller scale energy production, like rooftop solar, there is now a need to upgrade this system and allow for real time monitoring and a more dynamic power flow.

It would be ideal if the same methods used in transmission systems could also be applied to distribution systems, but unfortunately there are multiple differences that make this difficult:

* There are sections with three-phase, two-phase and single-phase power
* …

Text

Description automatically generated

DS: distribution systems

These complexities have resulted in specialized estimators for DSs:

### 4.7.1 State estimator based on power flow calculations

This is a power flow estimation done based on combining available measurements with pseudo measurements from load allocation algorithms. An iterative backwards/forwards sweep analysis is then performed until it returns calculated values close to the available monitored values

Steps: (pg. 93 or 103 in pdf)

Normal equation:

### 4.7.2 State estimator considering branch current flow as state variables

This approach uses the WLS method to obtain state variables, but the uses the complex current flow in rectangular form as the state variable rather than complex voltage!

Steps:

1. A backwards sweep is performed and initializes the state variables (current flow in the branches).  
   The nodal voltages are initialized by a forward sweep.
2. Obtain the updated current measurements
3. Obtain the correction of the state variables through the **normal equation** and update the state variable values  
   
4. Use the new state variable (current) values to update the nodal voltages through a forwards sweep
5. If convergence is not yet achieved, return to step 2

### 4.7.3 State estimator based on admittance matrix

This approach also uses equivalent measurements of complex voltage and current in rectangular form and all measurements are transformed into equivalent current.

The Jacobian matrix remains constant

Steps:

1. “An initial voltage value is assumed for all the buses (flat start or measured values, if available) and the available values of the power injection pseudo measurements.”
2. “Convert the flow and injection measurements of power and magnitude of voltage into equivalent current measurements.”
3. “Calculate the Jacobian and Gain matrices that will be kept constant in the estimation process.”
4. “Obtain the correction in the state variables of nodal voltages in the rectangular form through the normal Eq.”
5. “Convergence test: If the difference between the state vector of two consecutive iterations is less than the tolerance returns the estimated state. Otherwise, return to Step 4.”

### 4.7.4 Crucial challenges for DS state estimation

Although there already exist multiple methods for DS state estimation it is not yet clear what is the best solution, and many models suffer from beeing to specialized and can therefore not be generally applied to any DS.

These are some crucial challenges:

* **Distribution automation and smart meters:**
  + New equipment and communication standards
  + synchronization challenges between different types of measurements
* **Modeling of pseudo measurements:**
  + statistical models for pseudo measurements
  + incorporation of smart meters,
  + consideration of distributed generation in pseudo measurement modeling
  + Correlation between pseudo measurements
  + Modeling of the low voltage system
* **Wide area monitoring:**
  + Multiarea state estimation
  + Efficient numerical methods
  + Computational performance for real-time applications
* **Insertion of distributed generation:**
  + Medium voltage level (usually wind power)
  + Low voltage level (usually solar, photovoltaic, generation)
* Treatment of topology error caused by incorrect information on the status of switches and circuit breakers

## 4.8 Exercises

### Exercise nr 1

All the currents and nodal voltages of the **direct current system** in figure 4.4 are measured and shown in the measurement vector z, (current in Amps and voltage in volts)

Measurement vector  
z=[ I:1\_2 I :2\_1 I:1\_3 I:3\_1 I:2\_3 I:3\_2 V:1 V:2 V:3]T

z=[ 52, -49, -15, 16, -82, 80, 11, 6, 14]T

All available measurement has the standard deviation given by

Diagram, schematic

Description automatically generated

1. Determine the state vector (nodal magnitude voltages) by the WLS estimator (considering R, the covariance matrix equal to an identity matrix).

To obtain the best estimation of the state variables, , we will try to minimize the index given by:

is minimized when , and this can be expressed as:

Where is the Jacobian of , and is the vector with equations relating the state variables and the measurements…

CONSULT EXAMPLES!

1. Assuming that the errors of measurements I: 1–2 and I: 1–3 have variances 100 times smaller than the others, repeat item (a) using the WLS estimator

### Exercise nr 2

# Ch 5, Qualitative characteristics of measurement sets

(Pg. 111-168)

Qualitive characteristics of measurements generally relate to the observability and redundancy of a system. This is important to know both during the design and operation of a system.

The book considers modelling of a **single-phase power grid** and uses **complex nodal voltages** as state variables.

## 5.1 Observability analysis

A system is observable if all its state variables can be calculated from the available measurements and non-observable if this is not the case

When parts of a system in not observable there are two potential solutions:

* Identify the **observable islands** and calculate the state variables on these islands separately
* Restore observability by inserting critical **pseudo measurements** based on historical data, load forecasting, and generation data.

### 5.1.2 Definitions

There are multiple different types of observabilities:

**Algebraic observability:**To solve the **normal equation** it is necessary that the rank of the Jacobian matrix, , has a rank that is equal to or greater than the number of state variables in .

Thus, an EPS with n measurements is observabe if:

**Pθ and QV observability:**This type of observability is based on decoupling and busses and the fact that and sections of the Jacobian are more sensitive for calculating real and reactive power, respectively. Thus the and sections of the Jacobian can be neglected and:  
We have **Pθ observability** of an EPS with buses if  
  
and **QV observability** of an EPS with buses if  


**Numerical observability:**A system is numerically observable if you can iteratively find a solution for the normal equation, starting from a flat start. A prerequisite for beeing numerically observable is that the system is also algebraically observable.

**Topological observability:**The system must be a full rank spanning tree.

* Must have one measurement for each bus
* Is an injection measurement the net power that that bus (could be positive and negative)?

Topological methods are subject to combinational explosion because they must search for the entire spanning tree. However topological methods use **heuristic techniques** to improve the search process.

Question on page 105/115

**Spanning tree:** A tree that covers all busses of a system

**Full rank/observable spanning tree**: A spanning tree where at least one measurement can be assigned to each branch

**Observable branch:** power flow can be determined form available measurements

**Unobservable branch:** power flow cannot be determined form available measurements

**Irrelevant injection measurements:** Measurements that are incident to at least one unobservable branch (is it because we cannot know how much power goes into the unobservable branch?)

### 5.1.3 Methods

Topological methods:

* Based upon concepts of **graph theory**
* Requires new specific routines
  + Do not require calculations
  + Complex combinational nature

Numerical methods

* Use routines already available in state estimation software
* Are subject to numerical errors
* The most cited method (by Monticelli and Wu (1985)) relies on the triangular factorization of the gain matrix
* Many of these methods require a solution **algebraic equation system**

We will focus on methods that use triangular factorization

### Methods based on the triangular factorization of the Gain matrix and the concept of factorization paths

We consider the gain matrix of the linear WLS stat estimator, and the optimal solution of this estimator is calculated by:  
The terms are defined in chapter 4

##### Triangular factorization of gain matrix

* The observability of an EPS can be investigated through the relationship between reference angles and the triangular factorization of the matrix
* If the EPS is not completely observable the rank of the matrix will be less than .
  + If this is the case, the triangular factorization will give a null pivot on the diagonal, where

A picture containing chart

Description automatically generatedChart

Description automatically generated

##### Factorization path

* Factorization path is obtained from the **Lu factorization of a matrix** (the triangular factors responsible for factorization)
* The reactance of all busses is considered to be 1 pu

Diagram, schematic

Description automatically generatedA picture containing text, orange

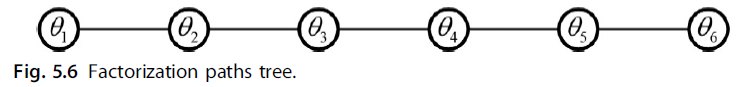
Description automatically generated

* LU decomposition the matrix gives the matrix of triangular factors
  + The factorization path of he non-zero elements is visualized by arrows

Table, calendar

Description automatically generated

* The rows of this matrix verify the flowing:  
  Text, letter

  Description automatically generated
* Resulting in the **factorization path tree**:  
  
* If a null pivot is encountered before the diagonal the factorization path will not reach this row, and it will indicate a discontinuous factorization path.
  + One or more separate factorization paths will then exist between the row and the row

##### Identification of observable island

* If the factorization path is discontinuous there will be separate factorization paths aka observable islands
* To find observable islands the injection measurements must be identified and disregarded to obtain a new gain matrix
  + This process is continued util there are no more injection measurements that that relate nodes of different islands.
* “The nodes belonging to each of the factorization paths found will constitute observable sub networks.”

##### Restoration of observability

* Adding, one at the time, pseudo injection measurements that relate/connect state variables of different paths.
* This will connect the factorization paths and eventually form a single path

#### Algorithm

1. Construct the Gain matrix, , with the available measurements, considering that:
2. Perform triangular factorization of
3. Identify the factorization path(s) of   
   **If one continuous path:** The EPS is observable, and the process is done  
   **If the path is discontinuous:** The EPS is not observable, continue through steps 4 through 6
4. Check for power injection measurements relating nodes of different paths  
   **If no offending power injections:** Each of the separate factorization paths already identify observable islands, go to step 6

**If there are offending power injections:** These power injections must be removed, go to step 5

1. Identify and discard the power injections that relate nodes of different factorization paths. Then, re-calculate the , and return to step 2
2. “Select a power flow injection pseudo measurement that will relate state variables of different factorization paths”  
   Note: we will only introduce one pseudo measurement at the time
3. Add the new pseudo measurement to the original measurements (nothing removed) and re-calculate (this is a little more complicated than just adding a row or column). Them, perform the triangular factorization and check the factorization path  
   **If one continuous path:** the EPS is not observable, and we are done  
   **If the path is discontinuous:** return to step 6 and repeat

### Method based on the triangular factorization of the Jacobian matrix and the concept of factorization paths

“This method is based on the triangular factorization of the Jacobina matrix with, if the system is observable, will give which relate measurements with equivalent state variables”

##### Restoration of observability

The restoration of observability is relatively straight forward:

1. Insert a new row in the matrix. This row should contain the equation associated the pseudo measurement
2. “Apply triangular factorization to the new row”  
   (meaning the matrix containing the new row?)
3. If a non-zero element now appears in the null pivot column, the pseudo measurement restored observability

##### Identification of observable islands

* It is necessary to include a new row with a 1 in the null pivot column
* To identify observable islands, it is necessary to eliminate power injection measurements that relate two different paths
* It is more computationally convenient to handle the transpose of the Jacobian matrix, so this is what will be done in the algorithm

#### Algorithm

1. Calculate the transposed Jacobian , considering:

If , introduce new columns filled with zeroes into

1. Page 132 (pdf)

## 5.2 Redundancy of analog measurements

Redundant measurements are measurements that if removed will not result in the system becoming un-observable. They can be viewed as extra measurements, but they are important to create a reliable system (and reduce errors?)

|  |  |
| --- | --- |
| Global redundancy: | This is the ratio between measurements and the amount of state variables to be estimated. Note that global redundancy does not assure observability |
| Critical Measurement (CM) | A measurement that, if removed, will make a system or island unobservable |
| Critical Set of Measurements (CSM) | Numeric definition: “Critical sets of measurements (CSMs) are those corresponding to the submatrices of the covariance matrix of residuals with a rank equal to 1.” |
| Topological definition:  A set of non-CMs were removing one of the set measurements in the set will result in the remaining measurements becoming CMs |
| P-critical sets of measurements | A set of measurements where removing all the set measurements will result in loss of observability, but where removing any amount of measurements, given that will not result in loss of observability. |
| Local redundancy level | A number given by which describes the redundancy at a given bus?  Note: the p-set used to represent a given bus is the p-set with the fewest measurements |

### 5.2.2 Methods Developed

Again, there are both topological and numeric methods for redundancy calculations. However, the book has focused mainly on topological methods.

The topological methods are in general based on the following properties:

* CMs have zero residual
* The main diagonal of the residual covariance matrix and residual sensitivity matrix corresponding to CMs are 0
* “Normalized residual of measurements from the same CSM are equal in module.”(pg. 141)
* “The correlation coefficient of measurements residuals from the same CSM are unitary.”  
  The correlation coefficient of the of the residuals of two measurements is given by:  
  : covariance matrix of residuals

London et al (2007) has developed a method for finding CMs and CSMs only based on the **triangular factorization of the Jacobian matrix** to get the matrix. However, this approach only works for completely observable systems.

What is the LU decomposition procedure?

### 5.2.3 Identification of the local redundancy levels through the matrix

: The Jacobian matrix of the EPS we are looking at

The matrix relates the active power measurements and the state variables of the ESP, and its row and columns correspond to power measurements (rows) and the state variables (columns).

CMs correspond to linearly independent rows of the matrix. However, this is initially difficult to analyze, so we manipulate the matrix:

For an EPS with buses and active power measurements, the system is algebraically observable if . We can then perform the change of base see below:

A picture containing diagram

Description automatically generated

Here the matrix has been split up into two components, manipulated, and renamed to , where:  
: Identity matrix of dimensions  
: Submatrix of dimensions composed of rows that are linearly dependent of   
Last column: only consisting of 0

**Basic measurements**: measurements that are needed for observability

**Supplementary measurements**: measurements that add to the redundancy of a system

#### Method description!!!!

It is easier to apply factorization processes when using the transpose of the matrix, and the method therefore uses , , and :

Diagram

Description automatically generated with low confidence

* To reduce rounding errors, the matrix is constructed with assigning the value of 1 to all line reactance (do we ignore the line resistance?)

##### Stages

1. Obtain and perform the **triangular factorization** to get the

The **Gauss elimination process** used for the triangularization is somewhat adapted to to be used on the

* 1. **Forward**: the non-zero elements of the lower triangle are made to 0 by linear combination of the rows

This method adopts the **columns deletion scheme**

* 1. **Diagonal** **operation**: make all the diagonal elements 1 by dividing the row of each diagonal by the value of the corresponding diagonal element
  2. **Backward**: The non-zero elements of the upper triangle are made to 0 by linear combination. This gives the identity matrix

1. Identify CMs (redundancy level = 0) and critical pairs and triplicates with only one basic measurement

This is done through the search for non-zero elements in the rows of

1. Identify critical pairs and triplicates involving more than on basic measurement
   1. Further triangular factorization of will reveal the critical pears and triplicates
2. Identification of measurements with a local redundancy level of 1 and 2 through the critical pear and triplicate measurements:
   1. **Redundancy level 1**: Non-critical measurements that appear in at least on critical pear
   2. **Redundancy level 2**: Non-critical measurements that do not appear in any critical pear but in at least on critical triplicate
   3. **Redundancy greater than 2**: The redundant measurements do not appear in any of the critical pears of triplicates

##### Algorithm

The or matrix is formed by first considering the flow measurements since the injection measurements can relate more than two variables and therefore result in more than one non-zero element in the or matrix columns associated with that measurement.

Steps:

1. Obtain with the available set of measurements
2. Perform the **forward** process until the pivot
3. Perform the **diagonal operation**
4. Perform the **backward** process
5. Check for non-zero elements in the rows of of the . From these measurements, identify CMs and critical pears and triplicates involving one basic measurement.
   1. **CM**: The row has only non-zero element on the diagonal of and only zeroes inside
   2. **Critical pair**: The row has only non-zero element on the diagonal of and one non-zero element inside
   3. **Critical triplicate**: The row has only non-zero element on the diagonal of and two non-zero element inside
   4. **Unconclusive**: There are more than two non-zero elements inside
6. Removing a basic measurement and reconstructing , reveals critical pears and triplicates with more than one basic measurement
   1. Note: if it is not necessary to search for critical pears or triplicates because the loss of any two measurements will make the system unobservable.
   2. Columns of the component of the matrix (that correspond to the basic measurements) are removed and replaced with columns from the before the new matrix is put through step 3 and 4
7. Identify the measurements with a local redundancy level of 1 and 2 through the critical pear and triplicate measurements:

### 5.2.4 Identification of CMs and CSMs from the matrix analysis

* A p-critical set with is also a CM
* A p-critical set with is also a CSM

Steps:

1. Obtain the measurements and find the CMs and critical pairs of measurements with only one basic measurement
2. Among the critical pairs, select the pairs who **do not** have any common supplementary measurements. These pairs are CSMs with only two measurements
3. Among the critical pairs, select croups wit at leas one common supplementary measurement. The measurements of this groups make a single CSM with more than two measurements  
   E.g.: If P:1 is a supplementary element, the critical pairs [P:1-2, P:1] and [P:2-3, P:1] combine to for the CSM [P:1, P:1-2, P:2-3]
4. If there are any non-critical basic measurements that do not belong to a CSM, the column corresponding to this measurement is eliminated form and a new is obtained. The basic measurements that are now identified as critical will construct a CSM together with the eliminated measurement  
   Repeat step 4 until all non-critical basic measurements have been analyzed

The example in the book is a little unclear about how it constructed the matrix , where it seems to have removed the P:1, P:1-2, and P:2-3 measurements without any further explanation

## 5.3 Qualitive characteristics of metering systems containing SPMs and SCADA measurements

For hybrid systems with buses the condition for algebraic observability is now:  
This is because there is no-longer a reference bus, and all voltage angles must therefore be known.

There are some additional complexities but in general the observability analysis is similar and need to confirm the absence null-pivots when processing the Jacobina matrix

## 5.4 Qualitive characteristics of metering systems containing SPMs and SCADA measurements

The measurement of an EPS is generally defined as reliable if there are no CMs, CSMs, or critical RTUs, and for hybrid systems there should be no critical PMUs.

Important notes:

* For an observable system all the EPS state variables can be determined
* GE processing cannot be done on CMs or CSMs
* I a system is free of no CMs, CSMs, critical RTUs, and critical PMUs, it will remain observable if up to at least two measurements are lost or even if all measurements transmitted by a TRU or PMU are lost

To assure observability the best approach would be to implement RTUs and PMUs everywhere, but this is costly.  
“Vigliassi et al. (2019) combines a multiobjective evolutionary algorithm based on subpopulation tables with the properties of the matrix”

RTU: remote terminal unit / loss of measurements

MSP: Metering System Planning

## 5.5 Update of the qualitive characteristics of measurements sets

To foresee the observability after the loss of a measurement it is important to know:

1. Whether the system remains observable
   1. If not, which pseudo measurements are needed to restore observability
2. What would be the new CMs and CSMs after the loss of the measurement

London et al. (2007) also developed a method this kind of analysis and fast updates when a measurement is lost. This revolves around an initial analysis, the **base case**, that considers all the measurements that should be available in the system and identifies CMs and CSMs. The a contineaous evaluation is done

1. If there is no loss of measurement, no analysis is needed
2. If a supplementary measurement is lost, it is indicated that the system remains observable, but a new analysis of criticality is performed based on an updated where the lost measurement has been removed. This can be done faster than the initial analysis thanks to the methods in 5.2.4
3. If a basic measurement is lost the system can have become unobservable.  
   To analyze observability the row corresponding to the lost measurement is removed from and the new matrix is checked for columns formed by only zeros.
   1. If the only zero-column is the last column of the matrix, the matrix is still observable. Further processing will then be done to obtain the new matrix and identify CMs and CSMs
   2. If there are multiple zero-columns the matrix is not observable  
      The necessary pseudo measurement is then identified form the triangular factors obtained fort he base case, it is restored, and a re-evaluation of the new matrix is done to and identify new CMs and CSMs

## 5.6 Qualitive characteristics considering the three-phase modeling of the electric network

Three-phase analysis becomes more relevant for distribution system, and this slightly complicates the observability analysis. However, the methods of London et al (2007) “was extended to allow the analysis of observability and redundancy of measurements from the triangular factorization of the Jacobian matrix of the three-phase WLS state estimator proposed in Zhong and Abur (2002).”

The main idea is that there must also be enough measurements to determine the voltage magnitude and angle of all phases.

# Ch 6, Gross error processing in measurements

Again, because the WLS state estimation process is the most widely implemented method, most GE processing methods are also based around this.

Situations where GE processing may fail:

* GEs of low-redundancy measurements. Meaning if a GE measurement is also a CMs or part of a CSMs
* Multiple GEs that interact (are correlated)
* GEs at leverage point measurements.  
  Leverage point are measurements that are highly influential and attract the convergence of the state estimation

## Vocab

**Detection**: The process determining whether a set of measurements contain any measurements with GEs

**Identification**: The process of determining with measurements have GEs

|  |  |
| --- | --- |
| Equation symbols and their significance | |
|  | Correction vector for measurements, used to reduce error |
|  | Estimation of the measurement correction value, |
|  | Projection of hat matrix, can say something about redundancy |
|  | Residual vector of all measurements |
|  | Residual covariance matrix,  Can determine the interaction of measurements |
|  | Residual sensitivity matrix. Represents the sensitivity of residuals with respect to their errors. Can be used to determine if two measurements are interacting |
|  | Number of measurements |
|  | number of state variables not including the reference bus angle |
|  | Test significance level, generally 5%=0.05 |
|  | Constant for chi-squared, |
|  | Probability distribution of |
|  | Normalized residual threshold for |
|  | The gamma function |
|  | Estimated error for measurement |
|  |  |

## index detection algorithm

1. Solve WLS state estimation and calculate the objective function:  
   :measurement I, :measurement function, :standard deviation of measurement
2. Determine/find the distribution table value that corresponds with a detection probability of 95% () and degrees of freedom. This value is defined as .

: number of measurements, :number of state variables (counting the reference bus?)

1. Test if   
   If yes: A gross error is suspected  
   If no: The measurements are assumed to be free of GEs

## Test Algorithm

1. Solve WLS state estimation and calculate the measurement residual vector:
2. Calculate the normalized residual
3. Find the measurement, , with the larges normalized residual,
4. Check if ­­ is greater than the threshold, , (), is normally sat to be   
   If yes: The measurement k is suspected to have a GE, precede to step 5  
   If no: The measurements are assumed to be free of GEs wright
5. Remove measurement and return to step 1

## Test Algorithm

1. Solve WLS state estimation to determine the state variables,
2. Calculate the normalized residual
3. Find the measurement, , with the larges normalized residual,
4. Calculate the estimated error for each measurement :
5. Check if the absolute value of the estimated measurement error exceeds the threshold.  
    is usually chosen to be 4.   
   If yes: The measurement k is suspected to have a GE, precede to step 6  
   If no: The measurements are assumed to be free of GEs
6. Remove measurement and return to step 1

## Hypothesis Test (HTI)

# Ch 7, The innovation methodology of error analysis in power systems

Pg. 193-219

# Vocabulary

## Abbreviations

### Ch 1-2

EPS: Electrical Power Source/System

PSSE: Power System State Estimation

SCADA: Supervisory Control and Data Acquisition

WLS: Weighted Last Square (the most popular static PSSE process)

WLAV: Weighted Last Absolute Value (more robust PSSE process)

WLMS: Weighted Last Mean of Squares (1st proposed statistically robust PSSE process that can deal with GEsk that are **points of leverage**. However, it requires combinational searches which makes it inconvenient for real time application for large EPS)

GEs: Gross Errors (generally defined as when a given measurement is 3 standard deviations away from its true value)

PE: Parameter Errors (errors in the information given about the electrical network)

TE: Topological Errors (incorrect information about switches and/or circuit breakers)

SPMs: Synchronized Phasor Measurements ()

PMU: Phasor Measurement Units

GPS: Global Positioning System

FASE: Forecasting-Aided State Estimated

PV:

PQ:

GU: Generator Unit

SC: Synchronized Condenser

TL: Transmission Line

TR: Transformer

Var: Reactive Power Meter

W: Active Power Meter

### Ch 3

cv

### Ch 4

DSs: Distributed Systems

EPS: Electrical Power Source/System

## Concepts

Ch2:

Emergency eliminations:

Corrective mode: when a power system control center executes a security control function.

Internal Systems: The section of the electrical grid that is within the supervision of a given company/authority

External Systems: The section of the electrical grid that is outside the supervision of a given company/authority that connects to the internal system borders

Operation Point:

Ch3:

**Nominal Voltage:** the voltage value that is assigned to a system without regarding uncertainties and variations. E.g.: the British power grid has a nominal voltage of 230 V, but the real value can vary from 216 V to 253 V. (Nominal voltage means the named voltage)

Nominal voltage is always given in line-to-line voltage.